

Math 213, Fall 2025, Oct. 29, 2025

Exam 2

Read Carefully, Present work in an organized way and show all work.

Write Neatly, Sketch Accurately

1. (8 points) Find the limit or show that the limit does not exist

a. $\lim_{(x,y) \rightarrow (1,1)} \frac{x^2 - y^2}{x - y}$

b. $\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{x^2 + y^2}$

2. (8 points) If $xy + z^3x - 2yz = 0$ defines z as a function of x and y , find $\frac{\partial z}{\partial x}$ at the point $(2, 8, 2)$.
3. (12 points) If $xy + z^3x - 2yz = 0$ defines a surface, find the equation of the tangent plane and the normal line (parametric equations) to the surface at the point $(2, 8, 2)$.
4. (14 points) Find the directional derivative of $f(x, y, z) = xe^y + z^2$ at $(2, \ln 2, 1/2)$ in the direction of $2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$. Find the maximum rate of change of f at $(2, \ln 2, 1/2)$ and the direction in which it occurs.
5. (20 points) Find the critical points of $f(x, y) = 6x - 8xy + 2y + 1$ and classify them. Then find the absolute maximum and minimum of f on the closed triangular plate bounded by the lines $x = 0$; $y = 0$ and $x + y = 2$.
6. (12 points) Use Lagrange Multipliers to find three real numbers whose sum is 9 and the sum of whose squares is as small as possible.
7. (12 points) Find the Taylor series *quadratic* expansion for $f(x, y) = e^x \cos y$ around $(0, 0)$ using the methods taught in this course. Use your approximation to approximate $e^{0.2} \cos 0.1$
8. (14 points) Sketch the region of integration for $\int_0^2 \int_{y/2}^1 \cos(x^2) dx dy$. Then reverse the order of integration and evaluate the integral.